Electron-drift driven ion-acoustic mode in a dusty plasma with collisional effects

R. Singh and M. P. Bora*
Institute for Plasma Research, Bhatt, Gandhinagar-382 428, India.

Instabilities of ion-acoustic waves in a dusty plasma with electron-drift, collisional, and dust charge fluctuations effects, have been investigated. The regimes are clearly marked out where the theory is applicable. The critical electron-drift velocity required to drive the instability is predicted. It is also shown that electron thermal conductivity and charged grains concentration enhance the growth of the ion-acoustic mode whereas ion-viscosity, ion-thermal conductivity, and dust charge fluctuations have a stabilizing effect.

I. INTRODUCTION

Dusty plasmas have acquired considerable importance[1] because of their applications to astrophysics and planetary physics[2], problems of plasma processing[3], and physics of strongly coupled systems[4]. Experimentally, one of the important areas of investigation is the study of low frequency ($\omega \leq \Omega_i$) fluctuations in a plasma with dust grains. In a recent experiment by Barkan et al.[5], it is shown that the phase velocity of ion-acoustic wave increases with the density of negatively charged dust grains and thus ion Landau damping becomes less severe in a plasma with charged dust grains[5] or with negative ions[6].

The ion-acoustic wave is a typical compressional mode in which ions provide the inertia and electrons bring in the pressure effects through their shielding cloud. In a magnetized plasma, it propagates with acoustic phase speed along the magnetic field lines. Earlier, the kinetic theory of the current-driven ion-acoustic instability, in a fully ionized, collisional[7,8] and collisionless[9] two-component plasma, has been investigated by various authors. In the limit of strong collisions (i.e. mean free path smaller than the wavelength), Coppi et al.[10] and Rognlien et al.[11] have used the two-fluid equations for an investigation of the current-driven ion-acoustic instability. Kulsrud and Shen[12] have investigated the effect of weak collisions on ion-acoustic wave using a Fokker-Plank model, while Stéfant[13] used kinetic theory and a collision integral of Bhatnagar-Gross-Krook type to show that ion-viscosity leads to a damping of the ion-acoustic instability, whereas, the electron-ion collisions tend to increase the growth rate. Recently, Rosenberg[7] has carried out the investigation of current-driven ion wave instability in a fully ionized collisionless dusty plasma using kinetic theory to predict the critical electron-drift velocity relative to the ions, required to drive the instability.

Interpretations of ion-acoustic wave excitation in a dusty plasma experiment have typically relied on the theories based on collisionless inverse electron Landau damping[10,11]. A realistic analysis of the experimental situation shows that in a typical experimental setup[5,6], the plasma density *may* be quite high $\sim 10^9$-$10^{10}\,\mathrm{cm}^{-3}$ when the plasma temperature is quite low. As an example, when plasma density $\sim 10^{10}\,\mathrm{cm}^{-3}$, with electrons and ions having approximately equal temperatures $T_e \sim T_i \sim 0.1\,\mathrm{eV}$, the electron-ion collision mean free path (λ_e) may be comparable or even smaller than the plasma scale length (L) along the magnetic field, i.e. $\lambda_e \lesssim L$. Under these conditions, the electrons no longer follow the Boltzmann relation and a collisionless theory of the electron-drift driven ion-acoustic instability is not justified. It is therefore important to re-examine this ion wave instability in a collisional dusty plasma. In these experiments cited above, however, ion-acoustic waves are launched by applying a sinusoidal voltage to a grid immersed in plasma rather than by drawing an electric current. Nevertheless, the effect of Coulomb collisions in presence of current driven ion-acoustic waves in such a plasma, as is considered in this work, remains to be seen in a laboratory experiment. In another experiment by Barkan et al.[14], current driven ion-acoustic waves in a dusty plasma are studied. But, this experiment was conducted at a relatively low plasma densities $\sim 10^6$-$10^7\,\mathrm{cm}^{-3}$.

In this paper, we investigate an electron-drift (or electron current) driven ion-acoustic mode in the presence of negatively charged dust grains. The dust charged grains are considered to be massive particles in a multicomponent plasma. This is valid when $a \ll d \ll \lambda_D$, where a is the average dust radius, d is the average distance between the dust particles, and λ_D is the plasma Debye length.

The paper is organized as follows. In Sec.II, the regimes of interest, where the theory is applicable are discussed. Sec.III deals with the linear instability theory. Sec.IV contains the conclusions.

*Permanently at *Physics Department, Gauhati University, Guwahati-781 014, India.*
Electronic mail : `mbora@gw1.dot.net.in`

II. REGIMES OF INTEREST

At first, we refer to regimes of interest where the present calculations are relevant. We consider a typical experimental situation such as considered in experiments by Barkan et al.[5,14] and Song et al.[6]. In such conditions, the typical parameters in these experiments are : potassium plasma (K^+) with $m_i/m_p \sim 40$, m_i and m_p being the ion and proton masses respectively, dust mass $m_d \sim 10^{-12}\,\mathrm{gm}$, gas pressure $\lesssim 10^{-5}\,\mathrm{Torr}$, the corresponding neutral (n) density $n_n \simeq 2\times 10^{11}\,\mathrm{cm}^{-3}$, equal electron ($e$) and ion ($i$) temperatures $T_e \sim T_i \sim 0.1\,\mathrm{eV}$, and dust grain temperature $T_d \sim 0.03\,\mathrm{eV}$ (assumed to be at room temperature). The plasma is confined radially by a magnetic field $B \sim 0.4\,\mathrm{T}$ and the plasma length along the magnetic field $L \sim 40$ cm with a radius $r_p \sim 2\,\mathrm{cm}$ for the plasma column. The characteristic frequency of the waves $f \sim$ 20-80 kHz which corresponds to $\omega = 2\pi f \sim$ 1-5$\times 10^5\,\mathrm{rad/s}$. The other plasma parameters are : electron-thermal velocity $c_e = (T_e/m_e)^{1/2} \sim 1.3\times 10^7\,\mathrm{cm/s}$, ion-thermal velocity $c_i = (T_i/m_i)^{1/2} \sim 5\times 10^4\,\mathrm{cm/s}$, the ion-acoustic speed $c_s = (T_e/m_i)^{1/2} \sim 5\times 10^4\,\mathrm{cm/s}$, the dust-thermal velocity $c_d = (T_d/m_d)^{1/2} \sim 2\,\mathrm{cm/s}$, the ion gyrofrequency $\Omega_i = eB/m_i c \sim 10^6\,\mathrm{s}^{-1}$, the ion Larmor radius $a_i = c_i/\Omega_i \sim 7\times 10^{-2}\,\mathrm{cm}$, the collision frequency of electrons and ions with the neutrals $\nu_{en} = n_n\sigma_{en}c_e \sim 1.3\times 10^4\,\mathrm{s}^{-1}$, $\nu_{in} = n_n\sigma_{in}c_i \sim 50\,\mathrm{s}^{-1}$ (where $\sigma_{in} \sim \sigma_{en} \sim 5\times 10^{-15}\,\mathrm{cm}^2$), with the corresponding mean free paths $\lambda_{en} \sim \lambda_{in} \sim 10^3\,\mathrm{cm}$. The e-i collision frequency (ν_{ei}) and i-i collision frequency (ν_{ii}) in the form presented by Braginskii[9] are calculated for the present parameters and are found to be as $\nu_{ei} = 1/\tau_e \sim 8.4\times 10^6\,\mathrm{s}^{-1}$ and $\nu_{ii} = 1/\tau_i \sim 2.8\times 10^4\,\mathrm{s}^{-1}$ and the mean free paths are $\lambda_e = c_e/\nu_{ei} \sim 1\,\mathrm{cm}$ and $\lambda_i = c_i/\nu_{ii} \sim 1\,\mathrm{cm}$, where $\tau_e = 3.5\times 10^4 T_{e0}^{3/2}/n\lambda$, $\tau_i = 3.0\times 10^6 (m_i/2m_p)^{1/2} T_{i0}^{3/2}/n\lambda$, and $\lambda = 23.4 - 1.15\log n + 3.45\log T \sim 9.5$ (for $T < 50\,\mathrm{eV}$) is the Coulomb logarithm.

From these calculations the following assumptions can now be made

(1) the collision mean free paths of electrons and ions are comparable to or even smaller than the plasma scale length along the magnetic field i.e. $\lambda_{e,i} < L$, so that we can make use of Braginskii's fluid equations,

(2) since $\omega < \nu_{ei}$, we can neglect the electron inertia,

(3) since $a_i < r_p$ and assuming a homogeneous plasma, the ion motion transverse to the magnetic field can be neglected (the polarization drift),

(4) since $\nu_{ei} \gg \nu_{en}, \nu_{ed}$ and $\nu_{ii} \gg \nu_{in}, \nu_{id}$, so the momentum and energy losses of electrons and ions to the neutrals and dust grains can also be neglected in the fluid equations,

(5) since the phase velocity of the observed ion-acoustic wave is much larger than the dust-thermal speed i.e. $v_p \gg c_d$, we can neglect the dust dynamics in the analysis. However, we shall incorporate the self-consistent fluctuation of charge on dust grains, which is shown to cause a damping effect on the ion-acoustic wave[15,16]. The charge fluctuation on the surface of the dust grains, which depends crucially on the dust size and the plasma density, is important when charging rate is comparable to the growth of the mode.

(6) we consider negatively charged grains as an additional charged plasma species of uniform massive particles similar to a multicomponent plasma. This remains valid as long as $a \ll d \ll \lambda_D$.

III. INSTABILITY ANALYSIS

We write down the basic equations in order to derive the linear dispersion relation of electron-drift driven ion-acoustic wave instability including the effects of dust charge fluctuations, electron and ion temperature perturbations, ion-viscosity etc. We consider a one-dimensional, plane, homogeneous plasma with low β ($\beta = 4\pi n T_e/B^2 \ll 1$ is the ratio of kinetic energy to magnetic energy). The resultant governing equations for electrons and ions are the Braginskii's fluid equations viz., equations of continuity, momentum, and energy.

We write down the equilibrium equations that define this specific regime of interest and confine ourselves to the parallel motions only. They are the ion and electron continuity equations

$$\frac{\partial n_i}{\partial t} + \nabla_\parallel \cdot (n_i \boldsymbol{u}_{i\parallel}) = 0, \tag{1}$$
$$\frac{\partial n_e}{\partial t} + \nabla_\parallel \cdot (n_e \boldsymbol{u}_{e\parallel}) = 0, \tag{2}$$

parallel electron and ion momentum equations

$$m_i n_i \frac{d\boldsymbol{u}_{i\parallel}}{dt} = -\nabla_\parallel p_i + e n_i \boldsymbol{E} + \mu_\parallel \nabla_\parallel^2 \boldsymbol{u}_{i\parallel}, \tag{3}$$
$$m_e n_e \frac{d\boldsymbol{u}_{e\parallel}}{dt} = -\nabla_\parallel p_e - e n_e \boldsymbol{E} + \boldsymbol{R}, \tag{4}$$

where $\boldsymbol{R}$ represents the momentum gained by the electrons through collision with the ions[9],

$$\boldsymbol{R} = \boldsymbol{R}_u + \boldsymbol{R}_T, \tag{5}$$
$$\boldsymbol{R}_u \simeq -m_e n_e \nu_{ei}(0.51\boldsymbol{u}_\parallel), \tag{6}$$
$$\boldsymbol{R}_T \simeq -0.71 n_e \nabla_\parallel T_e, \tag{7}$$
$$\boldsymbol{u}_\parallel = \boldsymbol{u}_{e\parallel} - \boldsymbol{u}_{i\parallel}. \tag{8}$$

In the above equations, $\boldsymbol{E}$ is the electric field driving the instability and $\mu_\parallel = 0.96 n_{i0} T_{i0}/\nu_{ii}$ is the parallel ion-viscosity coefficient[9]. In the ion momentum equation, however, we have neglected the collision term[10] in the limit $\boldsymbol{u}_{i\parallel} > (m_e/m_i)(\nu_{ei}/\omega)\boldsymbol{u}_\parallel$. We have further neglected the electron viscosity term in Eq.(4). The remaining equations are the energy equations[9],

$$\frac{3}{2}n_i\frac{dT_i}{dt}+p_i(\nabla_\parallel\cdot\boldsymbol{u}_{i\parallel})=\nabla_\parallel\cdot(\chi^i_\parallel\nabla_\parallel T_i), \tag{9}$$

$$\frac{3}{2}n_e\frac{dT_e}{dt}+p_e(\nabla_\parallel\cdot\boldsymbol{u}_{e\parallel})=\nabla_\parallel\cdot(\chi^e_\parallel\nabla_\parallel T_e) -0.71\nabla_\parallel\cdot(n_eT_e\boldsymbol{u}_\parallel), \tag{10}$$

where $\chi_{i\parallel}=3.9n_{i0}T_{i0}/m_i\nu_{ii}$ and $\chi_{e\parallel}=3.2n_{e0}T_{e0}/m_e\nu_{ei}$ are the parallel ion and electron-thermal conductivities, respectively. In the above equations, the terms $\sim\omega^{-1}_{ce,i}$ have neglecetd owing to the fact that $\omega<\omega_{ce,i}$.

We consider a small electrostatic perturbation and linearize the above equations. The linearized equations are,

$$\frac{\partial n_{i1}}{\partial t}+\nabla_\parallel(n_{i0}u_{i\parallel 1})=0, \tag{11}$$

$$\frac{\partial n_{e1}}{\partial t}+\nabla_\parallel(n_{e0}u_{e\parallel 1}+n_{e1}u_{e\parallel 0})=0, \tag{12}$$

where equilibrium and perturbed quantities are defined by the subscripts 0 and 1, respectively and $u_{e\parallel 0}$ is the zeroth order electron-drift velocity with respect to the ions and dust grains. The electrons drift with respect to the ions and dust particles, so that $\boldsymbol{u}_{i0}=\boldsymbol{u}_{d0}=0$. The ion motion along the magnetic field B is given by the linearized parallel momentum equation

$$m_in_{i0}\frac{\partial u_{i\parallel 1}}{\partial t}=-\nabla_\parallel(en_{i0}\varphi_1+n_{i0}T_{i1}+n_{i1}T_{i0}) +\mu_\parallel\nabla^2_\parallel u_{i\parallel 1}, \tag{13}$$

where φ_1 is the perturbed electrostatic potential. For $\omega<\nu_{ei}$, the parallel electron momentum equation becomes

$$0=\nabla_\parallel(en_{e0}\varphi_1-n_{e1}T_{e0}-n_{e0}T_{e1})-0.71n_{e0}\nabla_\parallel T_{e1} -0.51m_e\nu_{ei0}(n_{e0}u_{e\parallel 1}-n_{e0}u_{i\parallel 1}) -0.51m_en_{e0}u_{e\parallel 0}\nu_{e1}, \tag{14}$$

where $\nu_{e1}=\nu_{ei0}(n_{e1}/n_{e0}-3T_{e1}/2T_{e0})$ and the equilibrium drift velocity $u_{e\parallel 0}=-eE_{\parallel 0}/m_e\nu_{ei}$. The perturbed electron and ion temperature equations are given by

$$\frac{3}{2}n_{i0}\frac{\partial T_{i1}}{\partial t}+n_{i0}T_{i0}\nabla_\parallel u_{i\parallel 1}=\chi_{i\parallel}\nabla^2_\parallel T_{i1}, \tag{15}$$

$$\frac{3}{2}n_{e0}\left(\frac{\partial}{\partial t}+u_{e\parallel 0}\nabla_\parallel\right)T_{e1}+n_{e0}T_{e0}\nabla_\parallel u_{e\parallel 1}=\chi_{e\parallel}\nabla^2_\parallel T_{e1}-0.71n_{e0}T_{e0}\nabla_\parallel(u_{e\parallel 1}-u_{i\parallel 1}) -0.71u_{e\parallel 0}\nabla_\parallel(n_{e0}T_{e1}+n_{e1}T_{e0}), \tag{16}$$

Finally, we write the equations for dust charge fluctuations as given by Jana et al.[17,18] and the quasineutrality condition as

$$\left(\frac{\partial}{\partial t}+\eta\right)Q_{d1}=-|I_e|\left(\frac{n_{i1}}{n_{i0}}-\frac{n_{e1}}{n_{e0}}\right), \tag{17}$$

$$e(n_{e1}-n_{i1})+n_{d0}Q_{d1}=0, \tag{18}$$

where $Q_{d0}=eZ_{d0}$, Z_{d0} is the equilibrium charge number on the surface of the dust grains, $Q_{d1}=eZ_{d1}$, Z_{d1} is the charge fluctuation, $\eta=e|I_e|(T^{-1}_{e0}-W^{-1}_0)/C$ with $W_0=T_{i0}-e\phi_{f0}$, ϕ_{f0} is the potential at the dust surface, $|I_e|\sim I_i\sim en_{i0}\pi a^2c_s$ is the equilibrium electron (or ion) current at the dust surface, and $C\sim a$ being the grain capacitance. In writing Eq.(18), we have neglected the dust density fluctuations as the ion transit time ($\sim L/c_s$) is much shorter than the dust transit time[19] [$\sim L/c_{d*}$, $c_{d*}\sim(Z^2_{d0}n_{d0}T_{i0}/n_{i0}m_d)^{1/2}$] along the magnetic field.

It should, however, be noted that in writing the above equations, we have assumed a stationary background equilibrium. On the other hand, a non-stationary background equilibrium may lead to stabilization of the ion-acoustic instability in a low temperature collisional plasma[20].

We now take perturbations of the form

$$f(z,t)\sim f_1e^{-i(\omega t-k_\parallel z)} \tag{19}$$

and write Eqs.(11-18) as

$$\tilde{n}_i=\frac{k_\parallel u_{i\parallel 1}}{\omega}, \tag{20}$$

$$\tilde{n}_e=\frac{k_\parallel u_{e\parallel 1}}{(\omega-\omega_{\parallel 0})}, \tag{21}$$

$$k_\parallel u_{i\parallel 1}=\frac{k^2_\parallel c^2_s}{(\omega+i\hat{\mu}_\parallel)}\left(\tilde{\varphi}+\frac{\tilde{n}_i}{\tau}+\frac{\tilde{T}_i}{\tau}\right), \tag{22}$$

$$\tilde{n}_e+\left(1.71+i\frac{3}{2}\frac{\omega_{\parallel 0}}{\hat{\chi}_e}\right)\tilde{T}_e-\tilde{\varphi}=i\frac{k_\parallel}{\hat{\chi}_e}(u_{e\parallel 1}+u_{e\parallel 0}\tilde{n}_e-u_{i\parallel 1}), \tag{23}$$

$$\left(\frac{3}{2}\omega+i\hat{\chi}_{i\parallel}\right)\tilde{T}_i=k_\parallel u_{i\parallel 1}, \tag{24}$$

$$\left(\frac{3}{2}\omega - 2.21\omega_{\parallel 0} + i\hat{\chi}_{e\parallel}\right)\tilde{T}_e = k_\parallel(1.71u_{e\parallel 1} - 0.71u_{i\parallel 1}) + 0.71\omega_{\parallel 0}\tilde{n}_e, \tag{25}$$

where $\tau = T_{e0}/T_{i0}$, $\tilde{n}_e = n_{e1}/n_{e0}$, $\tilde{n}_i = n_{i1}/n_{i0}$, $\tilde{T}_i = T_{i1}/T_{i0}$, $\tilde{T}_e = T_{e1}/T_{e0}$, $\tilde{\varphi} = e\varphi_1/T_{e0}$ are the normalized variables with $\hat{\chi}_e \simeq k_\parallel^2 c_e^2/0.51\nu_{ei}$, $\hat{\mu}_\parallel = 0.96k_\parallel^2 c_i^2/\nu_{ii}$, $\omega_{\parallel 0} = k_\parallel u_{e\parallel 0}$, $\hat{\chi}_{e\parallel} = 3.2k_\parallel^2 c_e^2/\nu_{ei}$, and $\hat{\chi}_{i\parallel} = 3.9k_\parallel^2 c_i^2/\nu_{ii}$. We can now combine Eqs.(17) and (18) as

$$\left(1 + i\frac{Z_{d0}n_{d0}}{n_{e0}}\frac{\hat{I}_0}{(\omega + i\eta)}\right)\tilde{n}_e = \left(\frac{n_{i0}}{n_{e0}} + i\frac{Z_{d0}n_{d0}}{n_{e0}}\frac{\hat{I}_0}{(\omega + i\eta)}\right)\tilde{n}_i, \tag{26}$$

where $\hat{I}_0 = |I_e|/eZ_{d0}$ and $\lambda_D = (T_{e0}/4\pi n_{e0}e^2)^{1/2}$ is the plasma Debye length.

In the limit $\omega \sim \omega_{\parallel 0} < \hat{\chi}_{e\parallel}$ i.e. $3.2(k_\parallel \lambda_e)^2\nu_{ei}/\omega > 1$, Eq.(25) can be re-written, using Eq.(21), as

$$\tilde{T}_e = -i\frac{(1.71\omega - \omega_{\parallel 0})}{\hat{\chi}_{e\parallel}}\tilde{n}_e + i0.71\frac{\omega}{\hat{\chi}_{e\parallel}}\tilde{n}_i. \tag{27}$$

From Eqs.(20) and (24), we get

$$\tilde{T}_i = \frac{\omega}{\left(\frac{3}{2}\omega + i\hat{\chi}_{i\parallel}\right)}\tilde{n}_i. \tag{28}$$

Substituting Eqs.(22) and (28) into Eq.(20), we obtain

$$\left[1 - \frac{k_\parallel^2 c_i^2}{\omega(\omega + i\hat{\mu}_\parallel)} - \frac{k_\parallel^2 c_i^2}{(\omega + i\hat{\mu}_\parallel)\left(\frac{3}{2}\omega + i\hat{\chi}_{i\parallel}\right)}\right]\tilde{n}_i = \frac{k_\parallel^2 c_s^2}{\omega(\omega + i\hat{\mu}_\parallel)}\tilde{\varphi}. \tag{29}$$

Using Eqs.(20), (21), and (27), Eq.(23) can be written as

$$\left[1 - i\frac{\omega}{\hat{\chi}_e} - i1.71\frac{(1.71\omega - \omega_{\parallel 0})}{\hat{\chi}_{e\parallel}}\right]\tilde{n}_e - \tilde{\varphi} = -i1.75\frac{\omega}{\hat{\chi}_e}\tilde{n}_i. \tag{30}$$

In the limit of $\omega > \eta$, Eqs.(26), (29), and (30) can be combined to obtain the following dispersion relation,

$$\begin{aligned}\left[1 - \frac{k_\parallel^2 c_i^2}{\omega(\omega + i\hat{\mu}_\parallel)} - \frac{k_\parallel^2 c_i^2}{(\omega + i\hat{\mu}_\parallel)\left(\frac{3}{2}\omega + i\hat{\chi}_{i\parallel}\right)}\right]&\left[1 + i\frac{Z_{d0}n_{d0}}{n_{e0}}\frac{\hat{I}_0}{\omega}\left(1 - \frac{n_{e0}}{n_{i0}}\right)\right] \\ = \frac{n_{i0}}{n_{e0}}\frac{k_\parallel^2 c_s^2}{\omega(\omega + i\hat{\mu}_\parallel)}&\left[1 - i\frac{\omega}{\hat{\chi}_e} - i1.71\frac{(1.71\omega - \omega_{\parallel 0})}{\hat{\chi}_{e\parallel}}\right. \\ &\left. +\ i1.75\frac{\omega}{\hat{\chi}_e}\frac{n_{e0}}{n_{i0}}\left\{1 + i\frac{Z_{d0}n_{d0}}{n_{e0}}\frac{\hat{I}_0}{\omega}\left(1 - \frac{n_{e0}}{n_{i0}}\right)\right\}\right].\end{aligned} \tag{31}$$

For $T_{i0} \sim T_{e0}$ and $k_\parallel^2 c_e^2/\omega\nu_{ei} > 1$, it is obvious that $\omega > \hat{\mu}_\parallel, \hat{\chi}_{i\parallel}$ (i.e. $k_\parallel^2 c_i^2/\omega\nu_{ii} < 1$). We now write Eq.(31) as $\epsilon(k_\parallel, \omega) = 0$ and $\omega = \omega_r + i\gamma$ with $\omega_r > \gamma$. Taking $\varepsilon = \hat{\mu}_\parallel/\omega_r^{(0)}$ as an expansion parameter, the real part of ω, to the first order, can be written as,

$$\omega_r^2 \simeq (\omega_r^{(0)})^2 + 1.71k_\parallel^2 c_s^2\left(\frac{\hat{\mu}_\parallel}{\omega_r^{(0)}}\frac{n_{i0}}{n_{e0}}\frac{\omega_{\parallel 0}}{\hat{\chi}_{e\parallel}}\right), \tag{32}$$

where $\omega_r^{(0)}$ is given by

$$\omega_r^{(0)} = k_\parallel c_s\left(\frac{n_{i0}}{n_{e0}} + \frac{5}{3}\frac{T_{i0}}{T_{e0}}\right)^{1/2}. \tag{33}$$

The growth rate can now be written as

$$\begin{aligned}\gamma \simeq &-\frac{\hat{\mu}_\parallel}{2} - \frac{2}{9}\frac{k_\parallel^2 c_i^2}{\omega_r^2}\hat{\chi}_{i\parallel} - \frac{Z_{d0}n_{d0}}{2n_{e0}}\frac{n_{i0}}{n_{e0}}\frac{k_\parallel^2 c_s^2}{\omega_r^2}\hat{I}_0\left(1 - \frac{n_{e0}}{n_{i0}}\right) \\ &+ 1.71\frac{n_{i0}}{n_{e0}}\frac{k_\parallel^2 c_s^2}{2\hat{\chi}_{e\parallel}}\left(\frac{\omega_{\parallel 0}}{\omega_r} - 1 - \frac{Z_{d0}n_{d0}}{n_{i0}}\frac{\hat{\chi}_{e\parallel}}{\hat{\chi}_e}\right).\end{aligned} \tag{34}$$

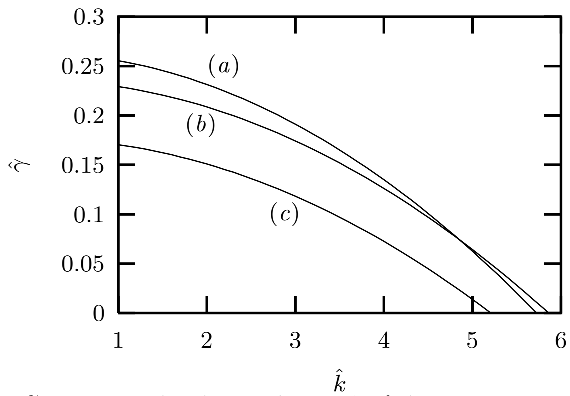

FIG. 1. Normalized growth rate $\hat{\gamma}$ of the ion-acoustic wave vs. normalized wave number $\hat{k}$ for different values of the parameter $\delta = n_{e0}/n_{i0}$. The curves labeled as (a), (b), and (c) are for $\delta = 1$ (no dust), 0.4, and 0.3, respectively. The other parameters are $\hat{u} = 6.0$, $\hat{I} = 0$, and $\hat{\lambda}_{e,i} = 0.01$.

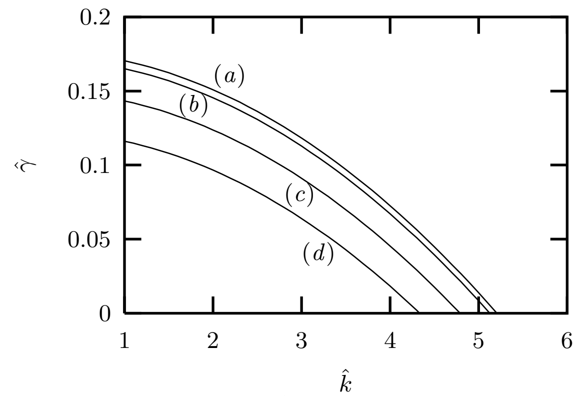

FIG. 2. Behavior of $\hat{\gamma}$ vs. $\hat{k}$ for (a) $\hat{I} = 0$, (b) 0.01, (c) 0.05, and (d) 0.1. Dust density parameter $\delta = 0.3$ and $\hat{\lambda}_{e,i} = 0.01$.

We note that, at the zeroth order, the phase velocity of the ion-acoustic wave is modified with a factor of $\left(\frac{n_{i0}}{n_{e0}} + \frac{5}{3}\frac{T_{i0}}{T_{e0}}\right)^{1/2}$. The presence of dust (decreasing ratio of n_{e0}/n_{i0}) leads to an increase in the phase velocity of the ion-acoustic wave.

In the expression for growth rate of the ion-acoustic wave Eq.(34), the collisional damping is represented by the first two factors, the parallel ion-viscosity and parallel ion-thermal conductivity. The third term causes damping due to charge fluctuation on the surface of the dust grains, which is proportional to the electron/ion current ($\hat{I}_0$) on the dust surface. We note that in absence of dust charge fluctuation (i.e. $\hat{I}_0 = 0$), increasing negatively charged dust density with respect to electron and ion density, results in an increase of the growth rate of the ion-acoustic instability. This can be seen from Fig.1, where we have plotted the normalized growth rate $\hat{\gamma} = \gamma L/c_s$ against normalized wave number $\hat{k} = k_\parallel L$, as obtained from the solution of the full dispersion relation Eq.(31), at different negatively charged dust concentration $\delta = n_{e0}/n_{i0}$. Note that $\delta = 0.3$ means 70% of the electronic charges is now carried by the massive dust particles and $\delta = 1$ means no negatively charged dust grains. The other parameters in Fig.1 are $\hat{u} = u_{e\parallel 0}/c_s = 6$, $\hat{\lambda} = \lambda_{i,e}/L = 0.01$, and $\hat{I} = \hat{I}_0 L/c_s = 0$. As in ion-acoustic instability without dust particles, the positive growth rate, here also, appears only when the drifting electron velocity $u_{e\parallel 0}$, exceeds several times the phase velocity. Therefore, the necessary condition for required electron-drift relative to ions and negatively charged dust grains to excite the ion wave instability is

$$
\begin{aligned}
u_{e\parallel 0} > c_s \left(\frac{n_{i0}}{n_{e0}} + \frac{5}{3}\frac{T_{i0}}{T_{e0}}\right)^{1/2} \Bigg\{ & \left(1 + \frac{Z_{d0} n_{d0}}{n_{i0}}\right) \\
& + (k_\parallel \lambda_e)^2 \frac{n_{e0}}{n_{i0}} \frac{T_{i0}}{T_{e0}} \frac{\nu_{ei}}{\nu_{ii}} \left[1.8 + \left(\frac{5}{3} + \frac{n_{i0} T_{e0}}{n_{e0} T_{i0}}\right)^{-1}\right] \\
& + \; 1.87 \left(\frac{Z_{d0} n_{d0}}{n_{i0}}\right)^2 \frac{n_{i0}}{n_{e0}} \left(\frac{n_{i0}}{n_{e0}} + \frac{5}{3}\frac{T_{i0}}{T_{e0}}\right)^{-1} \left(\frac{\pi a^2 \lambda_e n_{i0}}{Z_{d0}}\right) \left(\frac{m_i}{m_e}\right)^{1/2} \Bigg\},
\end{aligned} \tag{35}
$$

where we have used, only the zeroth order expression for ω_r. Note that the ion-viscosity and ion-thermal conductivity raise the critical electron current for the ion-acoustic wave to be unstable [the term in the square bracket in Eq.(35)], whereas the electron-thermal conductivity tends to increase the growth rate, as seen from Eq.(34).

The effect of increasing negatively charged dust density on growth rate of the ion-acoustic instability may be compensated by the presence of charge fluctuation on dust surface, which has a damping effect, as shown in Fig.2. In Fig.2 we have taken the dust density parameter $\delta = 0.3$. The other parameters are same as in Fig.1. We show in Fig.3, the behavior of $\hat{\gamma}$ versus δ for different values of dust current $\hat{I} = 0$, 0.05, 0.1, and 1.0. Other parameters chosen are $\hat{u} = 6$, $\hat{\lambda} = 0.01$, and $\hat{k} = 5.0$. The overall behavior is consistent with Fig.1 and Fig.2.

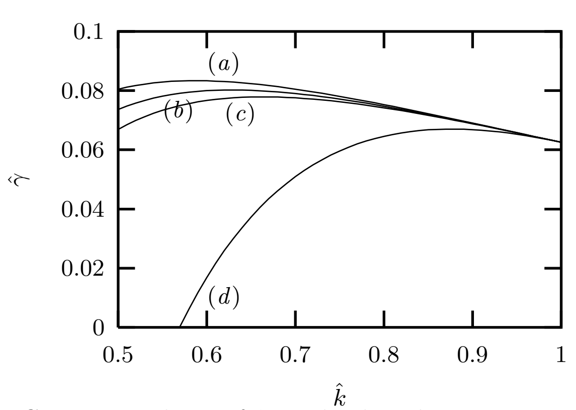

FIG. 3. Dependence of $\hat{\gamma}$ on the dust density parameter δ for different fluctuation level of the dust charge. (a) $\hat{I} = 0$ i.e. no dust charge fluctuation, (b) $\hat{I} = 0.05$, (c) $\hat{I} = 0.1$, and (d) $\hat{I} = 1.0$. Note the change in behavior of $\hat{\gamma}$ as the dust current increases. The normalized wave number $\hat{k} = 5.0$, $\hat{u} = 6.0$, and $\hat{\lambda}_{e,i} = 0.01$.

However, it is important to note that, although a decreasing δ causes peaking up of the growth rate, at any given wave number, a rapid dust charge fluctuation rate (larger $\hat{I}_0$) causes the wave to damp at higher dust concentration, which is opposite in behavior to the case of absence of dust charge fluctuation (Fig.1 and 2), as can be seen from all the curves in Fig.3.

For typical experimental parameters $n_{e0}/n_{i0} \sim 0.3$, $Z_{d0}n_{d0}/n_{i0} \sim 0.7$, $T_{e0} \sim T_{i0}$, and $\nu_{ei}/\nu_{ii} \sim (m_i/m_e)^{1/2}$, the necessary threshold condition for instability reduces to

$$u_{e\parallel 0} > 2.24 c_s \left[1.7 + \left(\frac{m_i}{m_e} \right)^{1/2} \left(0.7 k_\parallel^2 \lambda_e^2 + 0.6 \frac{\pi a^2 \lambda_e n_{i0}}{Z_{d0}} \right) \right]. \tag{36}$$

It is to be noted that the effects of charge fluctuation at the dust surface will tend to play an important role when the mean free paths are comparable to the plasma length along the magnetic field and the term

$$\frac{\pi a^2 \lambda_e n_{i0}}{Z_{d0}} \gtrsim 3 \left(\frac{m_e}{m_i} \right)^{1/2}. \tag{37}$$

Also notice that in high density and low temperature plasma limits, the effects ion-viscosity and thermal conductivity will be important, typically at wavelengths such that

$$k_\parallel \lambda_e \sim 1.5 \left(\frac{m_e}{m_i} \right)^{1/4}. \tag{38}$$

The condition (36) can be realized in typical laboratory situations[5]. For example, with an average size of dust grains to be few microns ($a \sim 10^{-4}$ cm) and a corresponding $Z_{d0} \sim 10^4$, the condition (36) yields

$$u_{e\parallel 0} \gtrsim 4 c_s, \tag{39}$$

for the excitation of the ion-acoustic instability, where we have taken the collisional parameters as $\lambda_{e,i}/L \simeq 0.01$ and $\hat{k} = k_\parallel L \sim 2$. The typical parallel electric field required for the corresponding threshold electron current is ~ 0.1 V/m.

IV. CONCLUSIONS

We have studied the ion-acoustic instability in a collisional dusty plasma with fluid equations, where dust grains are treated as massive and negatively charged component in a multicomponent plasma. The regimes are clearly marked out where the theory is applicable, especially in a relatively high density ($10^{10}\,\mathrm{cm}^{-3}$) and low temperature plasma where the effect of collisions cannot be neglected. While treating the negatively charged dust particles, we take into account the effect of charge fluctuation on the dust surface. We have shown that in such a plasma, which are routinely produced in laboratory, there is a significant impact of electron-ion collisions, even at large mean free paths (i.e. $\lambda_f \gtrsim L$). We have derived the threshold electron-drift velocity required to drive the ion-acoustic instability. It is shown that the electron-thermal conductivity and the dust charge concentrations reduce the threshold value of electron current for driving the ion-acoustic mode. In particular, the ion-viscosity and ion-thermal conductivity raise the threshold current. And a similar effect due to dust charge fluctuations is also found when the mean free paths are of the order of the plasma length.

ACKNOWLEDGMENT

It is pleasure to thank A. Sen for fruitful discussions. One of the authors, MPB, would also like to thank A. Sen for kind hospitality at IPR.

[1] *Physics of Dusty Plasmas, Proceedings of the VI workshop in Dusty Plasma*, edited by P. K. Shukla, D. A. Mendis, and

V. Chew (World Scientific, Singapore, 1996).

[2] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).

[3] D. B. Graves, IEEE Trans. Plasma Sci. **22**, 31 (1994).

[4] H. Ikezi, Phys. Fluids **29**, 1764 (1986).

[5] A. Barkan, N. D'Angelo, and R. L. Merlino, Planet. Space Sci. **44**, 239 (1996).

[6] B. Song, N. D'Angelo, and R. L. Merlino, Phys. Fluids **B3**, 284 (1991).

[7] M. Rosenberg, Planet. Space Sci. **44**, 229 (1993).

[8] N. D'Angelo, S. von Goeler, and T. Ohe, Phys. Fluids **9**, 271 (1966).

[9] S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965) Vol. 1, pp. 205.

[10] B. Coppi and E. Mazzucato, Phys. Fluids **14**, 134 (1971).

[11] T. D. Rognlein and S. A. Self, Phys. Rev. Lett. **27**, 792 (1971).

[12] P. M. Kulsrud and C. S. Shen, Phys. Fluids **9**, 177 (1966).

[13] R. J. Stéfant, Phys. Fluids **14**, 2245 (1971).

[14] A. Barkan et al., Phys. Lett. A **222**, 329 (1996).

[15] F. Li, O. Havnes, and F. Melandsø, Planet. Space Sci. **42**, 401 (1994).

[16] N. D'Angelo, Planet. Space Sci. **42**, 507 (1994).

[17] M. R. Jana, A. Sen, and P. K. Kaw, Phys. Rev. E **48**, 3930 (1993).

[18] R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).

[19] P. K. Kaw and R. Singh, Phys. Rev. Lett. **79**, 423 (1997).

[20] P. K. Kaw and A. K. Sundaram, Phys. Lett. **A38**, 355 (1972).